UDC 669.041

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DEVELOPMENT OF MATHEMATICAL MODEL AND ALGORITHM OF CALCULATIONS FOR HEATING THERMALLY MASSIVE BODIES IN FLAMING THERMAL FURNACES OF CHAMBER TYPE

(Report II)

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On the stage of decision of task, related to optimization of heating thermally massive bodies in the furnaces of chamber type, as a rule, it is set intercommunication between an action which controls, are temperatures of warming environment $\grave{O}_{\text{ïå÷}}$ and by the initial in-out parameters of process: by the temperature of surface of metal $T_{\text{ïîâ}}$, him by a mean temperature $\bar{T}_i$ and gradient of temperature between the superficial and middle layers of metal $\Delta\grave{O}_i$ which heat, and also sometimes its heating $T(x,\tau)$.

For practical tasks use exponential dependence in a kind:

$$T(x,\tau_{\text{ô}}) = \grave{O}_{\text{ïîâ}}(\tau_{\text{ô}})\cdot\exp\left[-\gamma(\tau_{\text{ô}})\cdot x\right], \tag{1}$$

where $T(x,\tau_{\text{ô}})$ - distribution of temperature in a metal on a co-ordinate x on the fixed moment of time $\tau_{\text{ô}}$; $T_{\text{ïîâ}}(\tau_{\text{ô}})$, $\gamma(\tau_{\text{ô}})$ - a temperature of metal surface which heat, and index of measure of exponent in the moment of time $\tau_{\text{ô}}$, accordingly.

In this connection distribution of temperature on a co-ordinate x is set by a function with one parameter $\tau_{\text{ô}}$, that influence, as

$$\gamma(\tau_{\text{ô}}) = \frac{1}{S}\cdot\ln\left[\frac{T_{\text{ïîâ}}(\tau_{\text{ô}})}{T_{\text{àîâ}}(\tau_{\text{ô}})+\Delta T_i}\right]. \tag{2}$$

For determination of dependence of metal surface temperature $T_{\text{ïîâ}}$ from the temperature of warming environment $\grave{O}_{\text{ïå÷}}$ carry out putting of function (1) to the maximum condition: $-\lambda_i \left.\frac{\partial T(x,\tau)}{\partial\tau}\right|_{x=0} = \alpha_\Sigma\cdot\left[T_{\text{ïå÷}}(\tau)-T_{\text{ïîâ}}(\tau)\right]$, where α_Σ – a total coefficient of heat emission, W/(m·K):

$$\grave{O}_{\text{ïîâ}}(\tau_{\text{ô}}) = \frac{\text{Bi}}{\text{Bi}\pm\gamma(\tau_{\text{ô}})\cdot S}\cdot T_{\text{ïå÷}}, \tag{7}$$

where a sign «±» determines the process of heating or cooling of metal, accordingly.

Coming from that answers every distribution of temperature of metal certain mean value of temperature $\bar{\grave{O}}$, it is possible to write down

$$\bar{T}\ \tau\ = \frac{T_{\ddot{\imath}\hat{\imath}\hat{a}}\ \tau\ \cdot \left\langle 1-\exp\left[\gamma\ \tau\ \right]\cdot S\right\rangle}{S\cdot\gamma\ \tau} \ . \qquad (10)$$

On the other hand, examining the process of heating of metal as accumulation of warmth in accordance with equation

$$\frac{d\bar{T}}{d\tau} = \frac{\alpha_{\Sigma}\cdot K_{\hat{o}}}{K_{\grave{\imath}}\ \cdot S\cdot\rho_{\grave{\imath}}\ \cdot c_{\grave{\imath}}}\cdot\ T_{\ddot{\imath}\hat{\imath}\hat{a}} - \bar{T}\quad , \qquad (11)$$

where $\hat{E}_{\hat{o}}$, $\hat{E}_{\grave{\imath}}$ are coefficients of form and massiveness of body, accordingly, - and marking permanent to heating time as $\grave{O}_{\grave{\imath}} = K_{\grave{\imath}}\ \cdot S\cdot\rho_{\grave{\imath}}\ \cdot c_{\grave{\imath}}\ /\ \alpha_{\Sigma}\cdot K_{\hat{o}}$ possible to give a temperature $\bar{T}$ as

$$\bar{T}\ \tau\ = \left[\bar{T}\ \tau_{\ddot{\imath}\hat{\imath}\div}\ - \grave{O}_{\grave{\imath}\hat{a}\div}\ \tau\ \right]\cdot\exp\ -\tau/T_{\grave{\imath}}\ + T_{\grave{\imath}}\ \tau\ \ . \qquad (12)$$

Accepting, that the value of middle temperature on the thickness of metal answers it’s to the mean level, and also equating right parts of equations (10) and (12), get an optimization mathematical model in a kind.

$$T_{\grave{\imath}\hat{a}\div}\ \tau\ = \frac{T_{\grave{\imath}}\ \tau\ \cdot\left\langle 1-\exp\left[-\gamma\ \tau\ \cdot S\right]\right\rangle - S\cdot\gamma\ \tau\ \cdot\bar{T}\ \tau_{\ddot{\imath}\hat{\imath}\div}\ \cdot\exp\ -\tau/T_{\grave{\imath}}}{S\cdot\gamma\ \tau\ \cdot\left[1-\exp\ -\tau/T_{\grave{\imath}}\ \right]} \ . \qquad (13)$$

Development over of management algorithm is brought to determination of moments by switching of the heat loading futnaces which calculate with use of formula

$$\tau = \grave{O}_{\grave{\imath}}\ \cdot\ln\left|\frac{\grave{O}_{\grave{\imath}\hat{a}\div} - \bar{O}\ \tau_{\ddot{\imath}\hat{\imath}\div}}{\grave{O}_{\grave{\imath}\hat{a}\div} - \left[\grave{O}_{\ddot{\imath}\hat{\imath}\hat{a}}\ /\ S\cdot\gamma\ \tau\ \right]\cdot\left\langle 1-\exp\left[-\gamma\ \tau\ \cdot S\right]\right\rangle}\right| \ . \qquad (14)$$

As initial data for a modeling set thermophysical parameters loads of metal, limitation on speed of its heating, temperature of furnace, initial and eventual distribution of temperature in relation to the cut of metal $T\ x,\tau_0$ and $T\ x,\tau_{\hat{e}}$.

On results calculations determine the temperature-sentinel mode at the action of which provide the set distribution of temperature in a metal during the minimum interval of time: $T\ x,\tau_0\ \Rightarrow \grave{O}\ \tilde{o},\tau_{\hat{e}}$.

An action which controls consists of $T_{\ddot{\imath}\hat{\imath}\hat{a},\max}$ in a range $0\le\tau\le\tau_1$ and from $T_{\ddot{\imath}\hat{\imath}\hat{a},\min}$ during time $\tau_1\le\tau\le\tau_1+\tau_2$. In this connection eventual distribution of temperature in a metal in the moment of time $\tau_1+\tau_2$ it is possible to examine as two components:

- change of its distribution on an action which controls, during time $0\le\tau\le\tau_1$;

- change of its distribution on an action which manages, during time $\tau_1\le\tau\le\tau_1+\tau_2$.

The index of measure of exponent $\gamma\ \tau_1+\tau_2$ is expected on a formula

$$\gamma\ \tau_1+\tau_2\ = \frac{1}{S}\cdot\ln\left[\frac{T_{\ddot{\imath}\hat{\imath}\hat{a},\max}}{T_{\hat{e}\grave{\imath}\acute{\imath}}}\right] \ . \qquad (17)$$

From equation (1) determine a temperature in points $x_1 = 0,25\,S$ and $x_2 = 0,75\,S$, and also exceeding the temperature ΔT set its eventual size:

$$\Delta T\ 0,25S\ = T_{\ddot{\imath}\hat{\imath}\hat{a},\max}\cdot\exp\left[-0,25\gamma\ \tau_1+\tau_2\ \right] - \grave{O}_{\hat{e}\grave{\imath}\acute{\imath}}\ 0,25S\ \ ; \qquad (19)$$

$$\Delta T\ \ 0,75S\ \ = T_{\ddot{\imath}\hat{\imath}\hat{a},\max} \cdot \exp\left[-0,75\gamma\ \ \tau_1 + \tau_2\ \ \right] - \grave{O}_{\hat{e}^3\acute{\imath}}\ \ 0,75S\ \ . \qquad (20)$$

The index of exponent $\gamma\ \ \tau_2$ is determined from correlation $\Delta T = 0,25\,S$ and $\Delta T = 0,75\,S$.

$$\gamma\ \ \tau_2\ \ = \frac{2}{S} \cdot \ln\left[\frac{\Delta T\ \ 0,25S}{\Delta T\ \ 0,75S}\right]. \qquad (21)$$